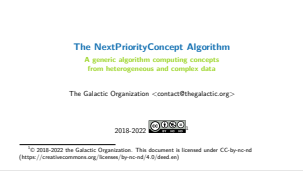
La Rochelle Université

The NextPriorityConcept Algorithm

A generic algorithm computing concepts from heterogeneous and complex data

The Galactic Organization <contact@thegalactic.org>



Motivations

- data scientist driven pattern mining;

GALACTIC stands for

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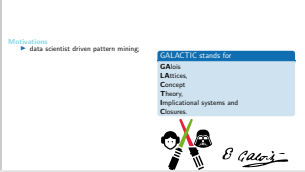

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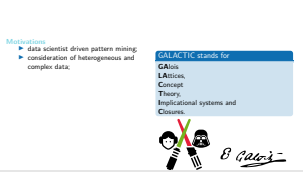

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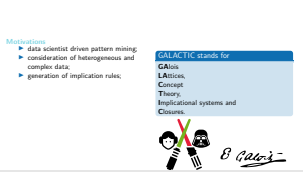

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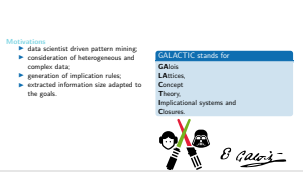

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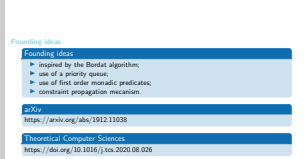
arXiv

https://arxiv.org/abs/1912.11038

Theoretical Computer Sciences

https://doi.org/10.1016/j.tcs.2020.08.026



Bordat algorithm as basis

A dual version of Bordat theorem

There is a bijection between the immediate predecessors of a concept (A, B) and the inclusion maximal subsets of the family:

$$\left\{ \beta(b) \cap A \ : \ b \in M \setminus B \right\}$$

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Concepts($\langle G, M, (\alpha, \beta) \rangle$)

```
begin
    top ← (G, α(G));
    Add top to a queue Q;
    while Q not empty do
        (A, B) ← Q.pop();
        produce (A, B);
        LP ← Immediate-Predecessors((A, B));
        forall (A′, B′) ∈ LP do
            Add (A′, B′) to Q
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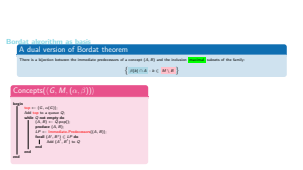

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Immediate-Predecessors((A, B))

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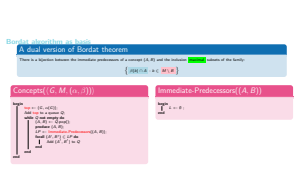

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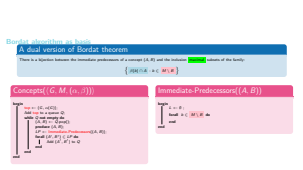

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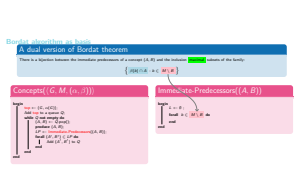

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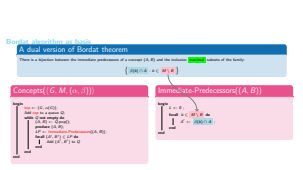

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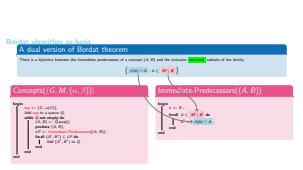

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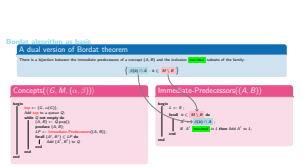

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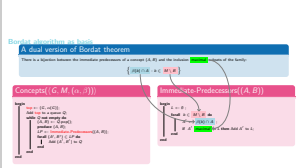

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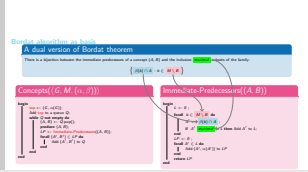

Selection of attributes: a strategy σ

Definition

Instead of all the possibles attributes in $M \setminus B$, we only consider some attributes, given by a strategy. A strategy σ is an application from 2^G to 2^M which associates a subset of selected attributes $\sigma(A) \subseteq M$ to every $A \subseteq G$.

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    if A' *maximal in* L **then** Add A' to L;
  end
  $LP \leftarrow \emptyset$;
  forall $A' \in L$ **do**
    Add $(A', \alpha(A'))$ to LP;
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Selected attributes P

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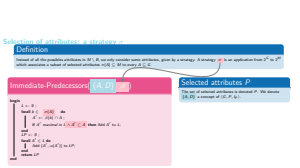

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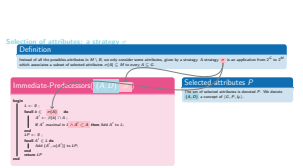

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Constraints

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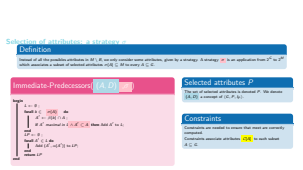
Constraints associate attributes $\mathcal{C}[A]$ to each subset $A \subseteq G$.



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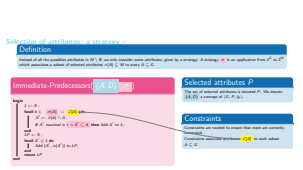

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    Compute the cross and residual constraints $C[A']$
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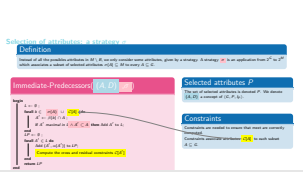

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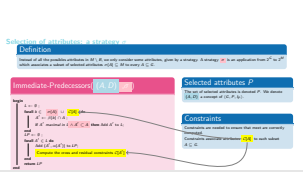
Constraints associate attributes $\mathcal{C}[A]$ to each subset $A \subseteq G$.



Selection of concepts: a priority queue

Concepts($\langle G, M, (\alpha, \beta)\rangle$)

begin
  top $\leftarrow (G, \alpha(G))$;
  Add top to a queue Q;
  while Q **not empty do**
    $(A,\ B\ ) \leftarrow Q$.pop();
    produce $(A,\ B\ )$;
    $LP \leftarrow$ Immediate-Predecessors$((A,\ B\ ))$;
    forall $(A',\ B'\ ) \in LP$ **do**
      Add $(A',\ B'\ )$ to Q;
    end
  end
end

Selection of concepts: a priority queue

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    produce $(A,\ B)$;
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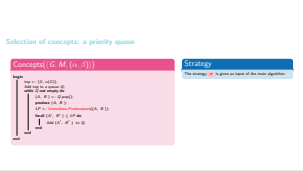
    forall $(A',\ B') \in LP$ **do**
      Add $(A',\ B')$ to Q;
    end
  end
end

Strategy

The strategy σ is given as input of the main algorithm.



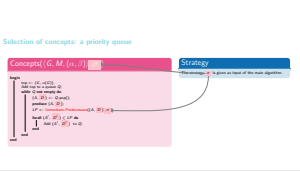
Selection of concepts: a priority queue

Concepts($\langle G, M, (\alpha, \beta)\rangle$, σ)

```
begin
    top ← (G, α(G));
    Add top to a queue Q;
    while Q not empty do
        (A, D) ← Q.pop();
        produce (A, D);
        LP ← Immediate-Predecessors((A, D), σ);
        forall (A′, D′) ∈ LP do
            Add (A′, D′) to Q;
        end
    end
end
```

Strategy

The strategy σ is given as input of the main algorithm.



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    end
end
```
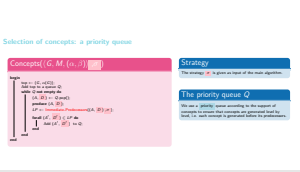

Strategy

The strategy σ is given as input of the main algorithm.

The priority queue Q

We use a priority queue according to the support of concepts to ensure that concepts are generated level by level, i.e. each concept is generated before its predecessors.



Selection of concepts: a priority queue

Concepts($\langle G, M, (\alpha, \beta)\rangle$, σ)

begin
  top $\leftarrow (G, \alpha(G))$;
  Add ($|G|$, top) to a priority ~~queue~~ Q;
  while Q **not empty do**
    $(A, D) \leftarrow Q$.pop();
    produce (A, D);
    $LP \leftarrow$ Immediate-Predecessors$((A, D), \sigma)$;
    forall $(A', D') \in LP$ **do**
      Add ($|A'|$,~~(A', D')~~) to Q;
    end
  end
end

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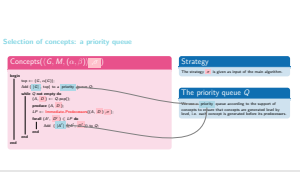

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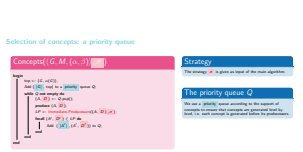


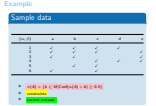
Example

Sample data

(α, β)	a	b	c	d	e
1	✓	✓	✓	✓	
2	✓	✓	✓		✓
3	✓	✓			✓
4			✓	✓	✓
5		✓	✓		
6	✓		✓		

- $\sigma(A) = \{b \in M | \text{Conf}(\alpha(A) + b) \geq 0.5\}$
- constraints
- current concept

(123456,)
abce



Example

Sample data

(α, β)	a	b	c	d	e
1	✓	✓	✓	✓	
2	✓	✓	✓		✓
3	✓	✓			✓
4			✓	✓	✓
5		✓	✓		
6	✓		✓		

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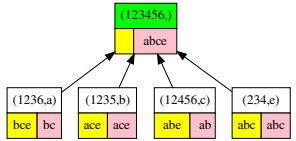

Example

Sample data

(α, β)	a	b	c	d	e
1	✓	✓	✓	✓	
2	✓	✓	✓		✓
3	✓	✓			✓
4			✓	✓	✓
5		✓	✓		
6	✓		✓		

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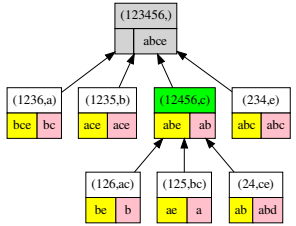

Example

Sample data

(α, β)	a	b	c	d	e
1	✓	✓	✓	✓	
2	✓	✓	✓		✓
3	✓	✓			✓
4			✓	✓	✓
5		✓	✓		
6	✓		✓		

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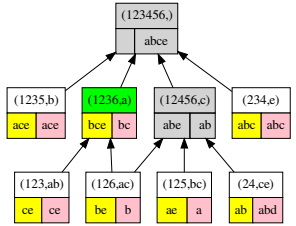

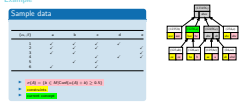

Example

Sample data

(α, β)	a	b	c	d	e
1	✓	✓	✓	✓	
2	✓	✓	✓		✓
3	✓	✓			✓
4			✓	✓	✓
5		✓	✓		
6	✓		✓		

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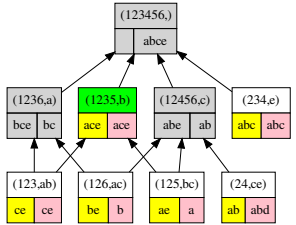

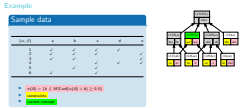

Example

Sample data

(α, β)	a	b	c	d	e
1	✓	✓	✓	✓	
2	✓	✓	✓		✓
3	✓	✓			✓
4			✓	✓	✓
5		✓	✓		
6	✓		✓		

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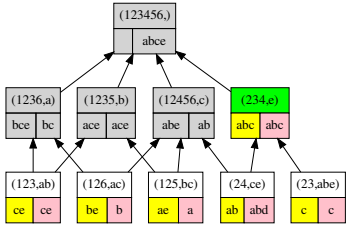

Example

Sample data

(α, β)	a	b	c	d	e
1	✓	✓	✓	✓	
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3	✓	✓			✓
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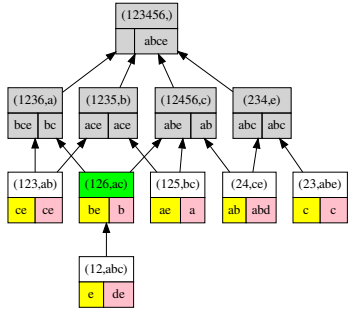

Example

Sample data

(α, β)	a	b	c	d	e
1	✓	✓	✓	✓	
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3	✓	✓			✓
4			✓	✓	✓
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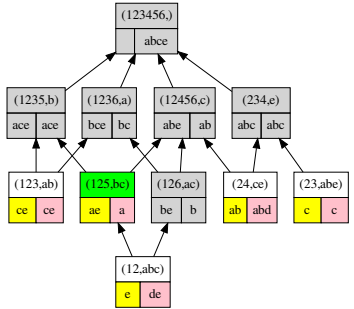

Example

Sample data

(α, β)	a	b	c	d	e
1	✓	✓	✓	✓	
2	✓	✓	✓		✓
3	✓	✓			✓
4			✓	✓	✓
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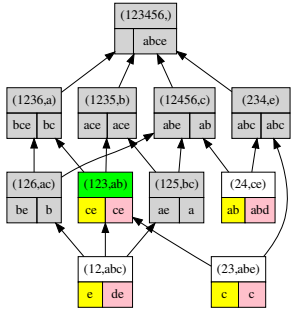

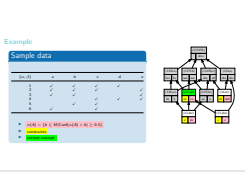

Example

Sample data

(α, β)	a	b	c	d	e
1	✓	✓	✓	✓	
2	✓	✓	✓		✓
3	✓	✓			✓
4			✓	✓	✓
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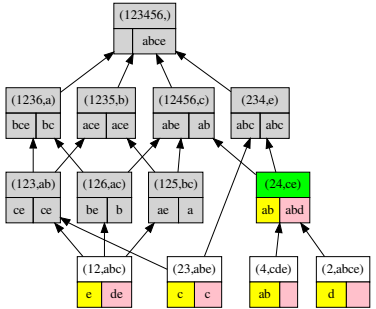

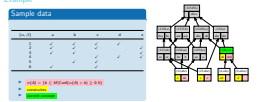

Example

Sample data

(α, β)	a	b	c	d	e
1	✓	✓	✓	✓	
2	✓	✓	✓		✓
3	✓	✓			✓
4			✓	✓	✓
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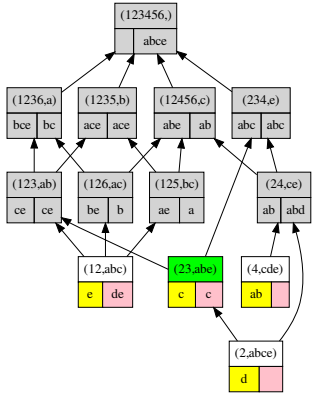

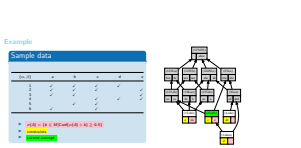

Example

Sample data

(α, β)	a	b	c	d	e
1	✓	✓	✓	✓	
2	✓	✓	✓		✓
3	✓	✓			✓
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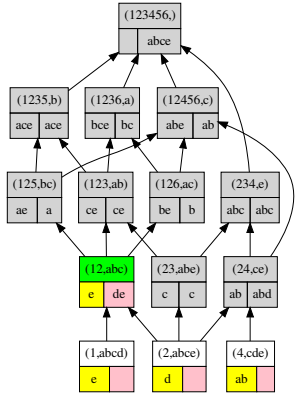

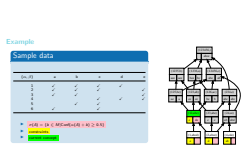

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Sample data

(α, β)	a	b	c	d	e
1	✓	✓	✓	✓	
2	✓	✓	✓		✓
3	✓	✓			✓
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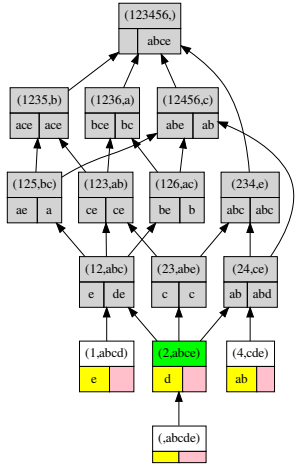

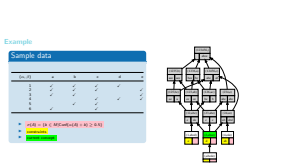

Example

Sample data

(α, β)	a	b	c	d	e
1	✓	✓	✓	✓	
2	✓	✓	✓		✓
3	✓	✓			✓
4			✓	✓	✓
5		✓	✓		
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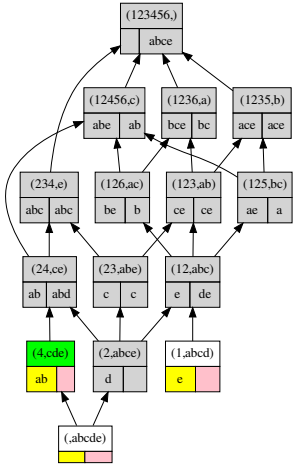

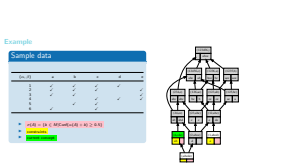

Example

Sample data

(α, β)	a	b	c	d	e
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2	✓	✓	✓		✓
3	✓	✓			✓
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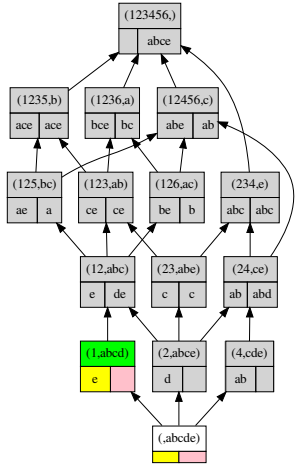

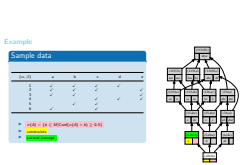

Example

Sample data

(α, β)	a	b	c	d	e
1	✓	✓	✓	✓	
2	✓	✓	✓		✓
3	✓	✓			✓
4			✓	✓	✓
5		✓	✓		
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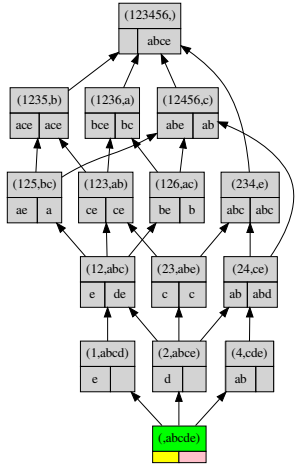

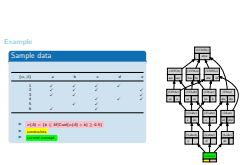

Example

Sample data

(α, β)	a	b	c	d	e
1	✓	✓	✓	✓	
2	✓	✓	✓		✓
3	✓	✓			✓
4			✓	✓	✓
5		✓	✓		
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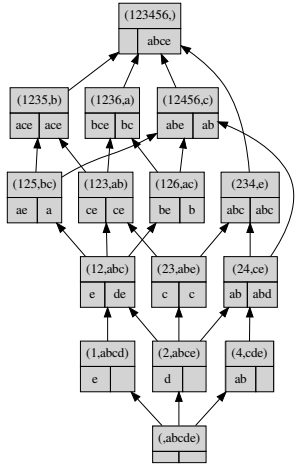
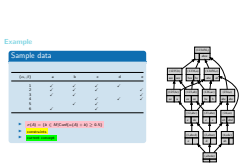


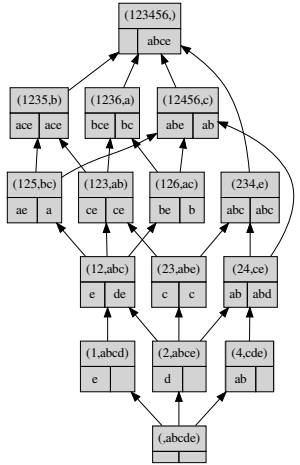


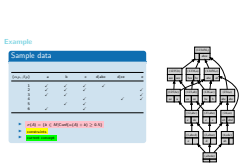
Example

Sample data

(α_P, β_P)	a	b	c	d|abc	d|ce	e
1	✓	✓	✓	✓		
2	✓	✓	✓			✓
3	✓	✓				✓
4			✓		✓	✓
5		✓	✓			
6	✓		✓			

- $\sigma(A) = \{b \in M | \text{Conf}(\alpha(A) + b) \geq 0.5\}$
- constraints
- current concept



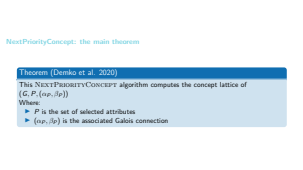


NextPriorityConcept: the main theorem

Theorem (Demko et al. 2020)

This NextPriorityConcept algorithm computes the concept lattice of $(G, P, (\alpha_P, \beta_P))$
Where:

- P is the set of selected attributes
- (α_P, β_P) is the associated Galois connection



Heterogeneous data as input

Concepts($\langle G, S, (S^i, \sigma^i, \delta^i)\rangle$)

begin
  top $\leftarrow$ $(G, \delta(G))$;
  Add $(|G|, \text{top})$ to a priority queue Q;
  while Q **not empty do**
    $(A, D) \leftarrow Q$.pop();
    produce (A, D);
    $LP \leftarrow$ Immediate-Predecessors$((A, D), \sigma, \delta)$;
    forall $(A', D') \in LP$ **do**
      Add $(|A'|, (A', D'))$ to Q;
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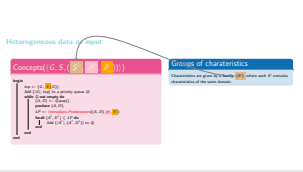
Groups of charateristics

Charateristics are given by a ~~family~~ (S^i) where each S^i contains charateristics of the same domain.



Heterogeneous data as input

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        end
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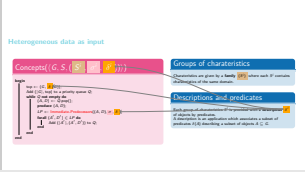
Charateristics are given by a **family** (S^i) where each S^i contains charateristics of the same domain.

Descriptions and predicates

Each group of charateristics S^i is provided with a **description** δ^i of objects by predicates.
A description is an application which associates a subset of predicates $\delta(A)$ describing a subset of objects $A \subseteq G$.



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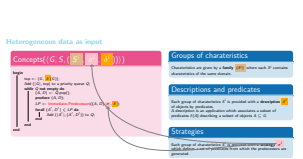
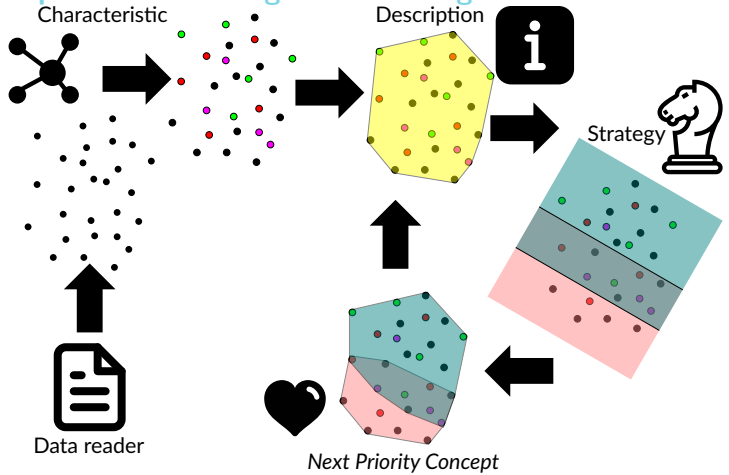
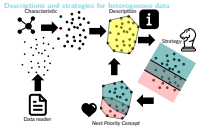
Strategies

Each group of charateristics S^i is provided with a **strategy** σ^i which defines a set of predicates from which the predecessors are generated.



Descriptions and strategies for heterogenous data





Descriptions and strategies for heterogenous data

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The strategy $\sigma(A)$ is composed of predicates describing a "cut" of the **generalized** convex hull of A from which the predecessors are generated.

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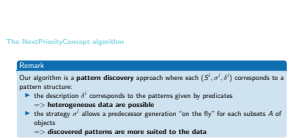
The NextPriorityConcept algorithm

Remark

Our algorithm is a **pattern discovery** approach where each $(S^i, \sigma^i, \delta^i)$ corresponds to a pattern structure:

- the description δ^i corresponds to the patterns given by predicates
  $=>$ **heterogeneous data are possible**
- the strategy σ^i allows a predecessor generation "on the fly" for each subsets A of objects
  $=>$ **discovered patterns are more suited to the data**



NextPriorityConcept

Theorem (Demko et al. 2020)

If each description δ^i verifies $\delta^i(A) \sqsubseteq \delta^i(A')$ for $A' \subseteq A$ then:
The NextPriorityConcept algorithm computes the concept lattice of $(G, P, (\alpha_P, \beta_P))$ where P is the set of predicates issued from the descriptions.

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